Dick Schaff Math Superbowl XLII
Level 3: Geometry Blitz

Directions:
1. Select the most correct answer for each question and mark it on your Scantron™ form.
2. Do not approximate for π unless instructed to do so.
3. N.O.T. stands for "None of These."

1. Let A and B be two points on the unit circle. Let C be the point where the tangent lines generated by A and B intersect. Let angle ACB have a measure of 90°. What is the area of triangle ACB?

a) 1/2 sq.unit b) π/4 sq.units c) 1 sq. unit d) π/2 sq.unit e) N.O.T.

2. Let A and B be two points on a circle with center P. Let C be the point where the tangent lines generated by A and B intersect. Let angle ACB have a measure of x°. What is the measure of angle APB?

a) 90° b) x° c) 2x° d) (180 – x)° e) N.O.T.

3. A sphere has volume k cubic units and surface area k square units. What is the radius of this circle?

a) 1 unit b) 3 units c) k units d) π units e) N.O.T.

4. A cube has volume k cubic units and surface area k square units. What is the edge length of this cube?

a) 1 unit b) 3 units c) k units d) π units e) N.O.T.

5. Some homeowners decided to install a small swimming pool. They initially dug a rectangular pit twenty feet long, nine feet wide, and six feet deep. How many cubic yards of dirt did they remove?

a) 40 b) 120 c) 360 d) 1,080 e) N.O.T.

6. Let A → B and B → C. If not-C is true, what else MUST be true?

a) A and B b) A or B c) B → A d) All of these e) N.O.T.

7. An equilateral triangle is inscribed within a unit circle with center P. What is the distance between P and the midpoint of any side of the triangle?

a) 1/3 unit b) 1/2 unit c) 1 unit d) π units e) N.O.T.

8. Problems 8 – 10 refer to the statement, "If you studied, then you did well."

8. Whish of the following is the negation of this statement?

a) You didn't study and you did well.
b) You didn't study or you did well.
c) You studied and you didn't do well.
d) You studied or you didn't do well.
e) N.O.T.

9. Which of the following is logically equivalent to the inverse of this statement?

a) You didn't study and you did well.
b) You didn't study or you did well.
c) You studied and you didn't do well.
d) You studied or you didn't do well.
e) N.O.T.

10. Which of the following is logically equivalent to the contrapositive of this statement?

a) You didn't study and you did well.
b) You didn't study or you did well.
c) You studied and you didn't do well.
d) You studied or you didn't do well.
e) N.O.T.

11. P, Q, and R are three distinct points on a unit circle. P and Q are endpoints of a diameter. What is the measure of angle PRQ?

a) 45° b) 60° c) 90° d) 135° e) N.O.T.

12. P, Q, and R are three distinct points on a unit circle. Arc PRQ has length $\pi/2$ units. What is the measure of angle PRQ?

a) 45° b) 60° c) 90° d) 135° e) N.O.T.

13. Which of the following CAN NOT be used to prove two triangles are congruent?

a) angle-side-angle
b) angle-side-side
c) side-angle-side
d) side-side-side
e) N.O.T.

14. A pizza parlor sells pizzas with 12", 14", 16", and 18" diameter. Which of the following provides the most pizza?

a) four 12" pizzas
b) three 14" pizzas
c) two 16" pizzas
d) one 18" pizza
e) N.O.T.

15. The radius of a right circular cylinder is increased by 10%, but the height is decreased by 20%. What is the volume of the new cylinder, as a percentage of the volume of the old cylinder?

a) 76.8% b) 96.8% c) 100% d) 129.6% e) N.O.T.

16. An empty ice cream cone is shaped like a right circular cone with a base diameter of 1.5 inches and a height of 4 inches. What is the volume of this cone?

a) $3\pi/4$ cu.in. b) $9\pi/4$ cu.in. c) 3π cu.in. d) 9π cu.in e) N.O.T.

17. A hemispherical scoop of ice cream is added to the cone from Problem 16. The ice cream and the cone have the same radius. What is the volume of the ice cream?

a) $9\pi/64$ cu.in. b) $9\pi/32$ cu.in. c) $9\pi/16$ cu.in. d) $9\pi/4$ cu.in e) N.O.T.

18. All mathletes are students. All students love ice cream. Alex loves ice cream. What can we conclude?

a) Alex is a mathlete
b) Alex is not a mathlete
c) Alex is a student
d) Alex is not a student
e) N.O.T.

19. Triangles ABC and DEF are congruent. Which of the following is NOT true?

a) angle A is congruent to angle D
b) angle B is congruent to angle E
c) line segment AB is congruent to line segment DE
d) line segment AC is congruent to line segment DF
e) N.O.T.

20. A square has perimeter 40 cm. What is the area of this square?

a) 100 mm^2 b) 1000 mm^2 c) 10000 mm^2 d) 1600 cm^2 e) N.O.T.

21. What is the maximum number of 1" by 3" by 4" rectangular boxes that will fit in a 2' by 3' by 3' crate?

a) 96 b) 216 c) 1,152 d) 2,592 e) N.O.T.

22. Alex is 4' 2" tall. One day Alex casts a shadow 1' 8" long. A nearby street sign casts a shadow 4' 2" long. How tall is the street sign?

a) 1' 8" b) 6' 8" c) 10' 5" d) 12' 5" e) N.O.T.

23. A dozen ½-inch diameter spherical glass marbles are dropped into a half-full glass of water. The glass is in the shape of a right circular cylinder, and dropping in the marbles raises the level of the water ¼ of an inch. What is the radius of the glass?

a) 1 inch b) 1.5 inches c) 2 inches d) 3 inches e) N.O.T.

24. Let C_1 and C_2 be circles. The ratio of the area of C_1 to the area of C_2 is 9 : 4. What is the ratio of the circumference of C_1 to the circumference of C_2

a) 4 : 9 b) 3 : 2 c) 9 : 4 d) 81 : 16 e) N.O.T.

25. Which of the following is a definition of a parallelogram?

a) A quadrilateral with four right angles
b) A quadrilateral with four equal sides
c) A quadrilateral with a single pair of parallel sides
d) A triangle with two equal sides
e) N.O.T.

26. A circular window with a diameter of four feet is surrounded by a 3-inch wide frame of uniform width. What is the area of the frame?

a) $\pi/16$ sq.ft b) $17\pi/16$ sq.ft. c) $33\pi/16$ sq.ft d) 21π sq.ft e) N.O.T.

27. A 4 m tall pyramid has a square base 6 m wide on a side. What is the volume of this pyramid?

a) 32 m^3 b) 48 m^3 c) 96 m^3 d) 144 m^3 e) N.O.T.

28. A 4 m tall pyramid has a square base 6 m wide on a side. What is the surface area of this pyramid (including the base)?

a) 48 m^2 b) 60 m^2 c) 84 m^2 d) 96 m^2 e) N.O.T.

29. How many vertices does a tesseract have?

a) 8 b) 16 c) 24 d) 32 e) N.O.T.

30. Let line segments AB and DE have the same length, let line segments AC and DF have the same length, and let angles ABC and DEF have the same measure. What can we say about triangles ABC and DEF?

a) They are both acute
b) They are both obtuse
c) They are congruent
d) They are similar
e) N.O.T.

31. A chord of length 24 units is drawn in a circle of radius 13 units. What is the distance from the chord to the center of the circle?

a) 5 units b) 10 units c) 11 units d) 13 units e) N.O.T.

32. Consider regular octagon ABCDEFGH. Remove vertex H, and use the remaining points to form heptagon ABCDEFG. What is the measure of angle BAG?

a) 90° b) 112.5° c) 135° d) 157.5° e) N.O.T.

33. Quadrilateral ABCD is inscribed within a circle. Let the measure of angle A be x°. What is the measure of angle C?

a) x° b) $(90 - x)^\circ$ c) $(180 - x)^\circ$ d) $(90 + x)^\circ$ e) N.O.T.

34. A factory manufactures cubical (six-sided) dice and ships them to game companies. A tightly-packed cubical shipping box contains 512 of these dice. How many of these dice are touching at least one face of the box?

a) 128 b) 216 c) 296 d) 384 e) N.O.T.

35. Which of the following regular polygons will NOT tessellate the Euclidean plane?

a) triangle b) quadrilateral c) pentagon d) hexagon e) N.O.T.

36. What is the area of a circle with diameter D units long?

a) $0.25\pi D^2$ sq.units b) $0.5\pi D^2$ sq.units c) πD^2 sq.units d) πD sq.units e) N.O.T.

37. A pentagon has interior angles measuring $2x^\circ$, $3x^\circ$, $3x^\circ$, $3x^\circ$, and $4x^0$. What is the measure of the largest of these angles?

a) 36° b) 72° c) 108° d) 120° e) N.O.T

38. A dog is leashed to a tree. The trunk of the tree has a diameter of eight inches. The leash is twenty feet long. The dog runs clockwise around the tree until the entire leash is would tightly around the trunk. How many complete revolutions did the dog make?

a) 8 b) 9 c) 10 d) 11 e) N.O.T.

39. A square piece of cardboard is twelve inches long on a side. Squares of side length x are cut from the corners, and the sides are folded up to make an open-topped box. What is the volume of this box?

a) 144 sq.in. b) 144x sq.in. c) $x(12 - x)^2$ sq.in. d) $4x(6 - x)^2$ sq.in. e) N.O.T.

40. Angles A and B are complementary. Angle A has a measure of 29°. What is the measure of angle B?

a) 29° b) 71° c) 119° d) 151° e) N.O.T.

41. Let triangles ABC and DEF be similar, and let the length of AB be twice the length of DE. If the area of triangle ABC is 0.5K, what is the area of triangle DEF?

a) 0.25K b) 0.5K c) 1.0K d) 2.0K e) N.O.T.

42. In a two-column geometric proof, which of the following could NOT be a reason to justify the claim that angles A and B are congruent?

a) Given
b) Vertical angles are congruent
c) Alternate interior angles are congruent.
d) Corresponding parts of congruent triangles are congruent.
e) N.O.T.

43. In a two-column geometric proof, which of the following could NOT be a reason to justify the claim that line segments AB and CD are congruent?

a) Given
b) Side-Side-Side Congruence
c) Opposite sides of a parallelogram are congruent.
d) Corresponding parts of congruent triangles are congruent.
e) N.O.T.